Mathematic Principles behind the Scheme of Setting Base Stations’ Locations by Measuring Distances

1. Abstract

This article is aimed at the base station placement problem in the UWB positioning scheme, using mathematic methods to prove the strategy of setting base stations’ locations by measuring distances between each two UWB modules, including base stations and tags.

1. Description of the Base Station Placement Strategy

The previous strategy of setting base stations is to set the positions in the host computer program, then place the base stations to the set positions in the field. The main problem is that there will be errors between the actual positions of the base stations and the set positions. So, we use a new strategy to solve this problem. We place the base stations to the set positions in the field first, then initialize their positions in computer program by using base stations themselves to measure distances between each two base stations. This method avoids the error from placing base stations, so that it can increase the accuracy of the localization system.

1. Mathematical Derivation Under 2-Dimensional Situation

Denote , and are base stations, is a tag. Through transmission between UWB modules, we know the distances between every two points, which means the length of , , , , , are our initial constraints.

To simplify the problem, we assume that is on the original point and the line is on the X axis. So, the coordinate of is (0,0), the coordinate of is (0,) (see Figure 1).

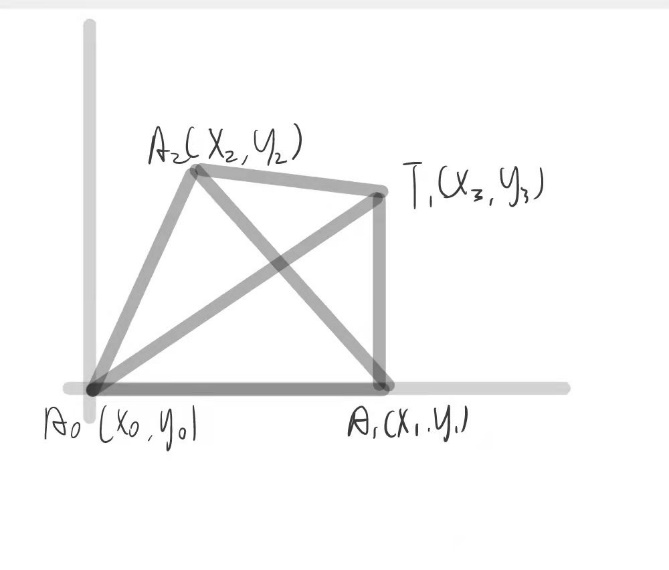


Figure 1.

First, we know that

After elimination, one obtains

Hence , Consequently, the remain coordinates are given by  
However, we cannot use the coordinates directly, because we cannot make sure that we use the most accurate data of the distances between every two points. So, we need to do error deduction to make the distances used be closest to the true value.

Given , , , , ,

Estimate , , , ,

Find error

Our might now define the objective function

Which we will attempt to minimize. As an initial guess, let us use the calculation above which is in ideal state:

We know that

Here is for learning rate.

Where the Jacobian matrix is given by

We calculate

Thus

can be calculated.

And can be calculated.

Now, a suitable must be found such that

This can be done with any of a variety of line search algorithms. One might also simply guess a , which gives .

Evaluating the objective function at this value, yields

The decrease from to the next step’s value of is a sizable decrease in the objective function. Further steps would reduce its value further, until an approximate solution to the system was found.